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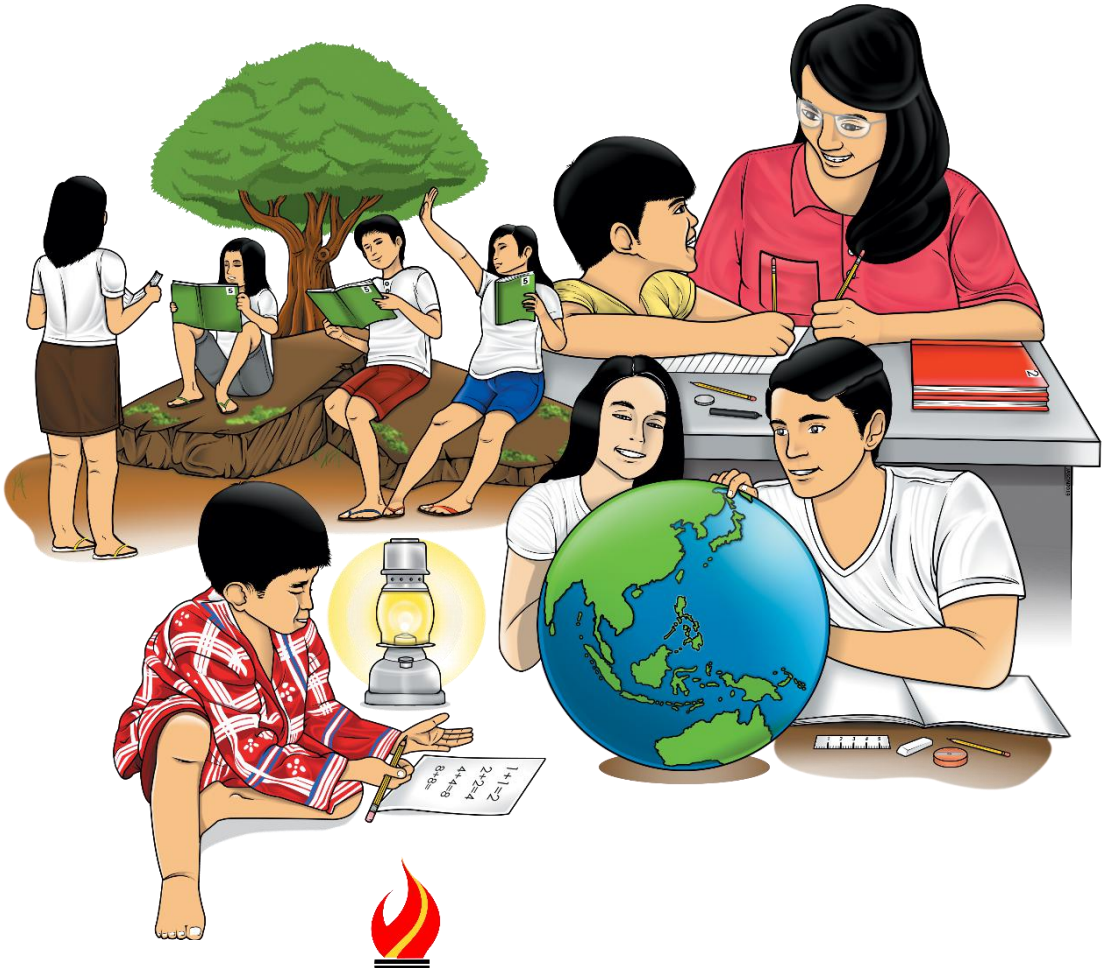
Mathematics

Quarter 1 – Module 1:
Generating Patterns



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Development Team of the Module

Writer's Name: Melchor B. Ticag
Editor's Name: Laila B. Kiw-isen
Reviewer's Name: Bryan A. Hidalgo
Management Team:
May B. Eclar, PhD
Marie Carolyn B. Verano
Carmel F. Meris
Ethielyn E. Taqued
Edgar H. Madlaing
Soraya T. Faculo
Francisco C. Copsiyan

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Telefax: (074) 422-4074
E-mail Address: car@deped.gov.ph

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Quarter 1 – Module 1:

Generating Patterns

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Introductory Message

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

For the facilitator:

This module is the first learning competency in our Mathematics 10 curriculum standards hence mastery of the skills set for your students is significant for them to have a smooth progress in the succeeding lessons. This learning material is designed to equip the students with essential knowledge about generating patterns as an introduction to sequences and patterns. Please have patience in assisting the learners accomplish this module.

For the learner:

This module is the first learning competency in our Mathematics 10 curriculum standards hence mastery of the skills is significant for you to have a smooth progress in the succeeding lessons. This learning material serves as a bridge from your grade 8 reasoning skills to the introduction of sequences and patterns. By doing the prepared activities, it is expected from you to develop your skill in generating patterns. Please read completely the written texts and follow the instructions carefully so that you will be able to get the most of this learning material. We hope that you will enjoy learning.

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
	**What I need to know**	This will give you an idea of the skills or competencies you are expected to learn in the module.
	**What I know**	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
	**What's in**	This is a brief drill or review to help you link the current lesson with the previous one.
	**What's new**	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
	**What is it**	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.

	**What's more**	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	**What I have Learned**	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
	**What I can do**	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
	**Assessment**	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	**Additional Activities**	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
	**Answer Key**	This contains answers to all activities in the module.

At the end of this module you will also find:

References This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

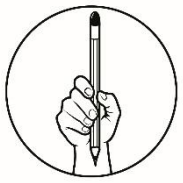
We hope that though this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in mind. It is here to indulge you in discovering and generating patterns. The scope of this module permits it to be used in many different learning situations. Recognizing and extending patterns are important skills needed to the learning of concepts related to sequences. The arrangement of the lessons follows the standard sequence of the course. But the pacing in which you read and answer this module is dependent on your ability.

Primarily, the scope of this module is to develop your skill in generating patterns. While going through this module, you are expected to:

1. generate pattern from a given succession of objects, numbers, letters, or symbols;
2. find the nth term of a sequence; and
3. write the rule for the nth term of a sequence.



What I Know

Let us determine how much you already know about generating patterns. If you answer all the test items correctly in this pretest, then you may skip studying this learning material and proceed to the next learning module.

DIRECTION: Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1. Which of the following does not show a sequence?
   A) K to 12 Curriculum
   B) Months of a year
   C) Body temperature
   D) Counting numbers

2. Which of the following is a finite sequence?
   A) negative even numbers
   B) even numbers below 100
   C) even numbers greater than 1
   D) even numbers between 10 and 50

3. What is the 9^{th} term in the sequence $-1, 4, -9, 16, -25, ..?$
   A) 64 B) -81 C) -64 D) 81

4. Find the first four terms of the sequence $a_n = 5n - 1$.

A) 5, 10, 15, 20
B) 6, 11, 16, 21
C) 4, 9, 14, 19
D) 5, 9, 13, 17

5. What is the 20th term of the sequence $a_n = \frac{(-1)^n}{n^2}$?

A) $\frac{1}{400}$ B) $-\frac{1}{400}$ C) $\frac{20}{400}$ D) $-\frac{20}{400}$

6. What rule will correctly describe the sequence: 2, 5, 10, 17, 26, 37, …?

A) $n + 1$ B) $2n$ C) $2n + 1$ D) $n^2 + 1$

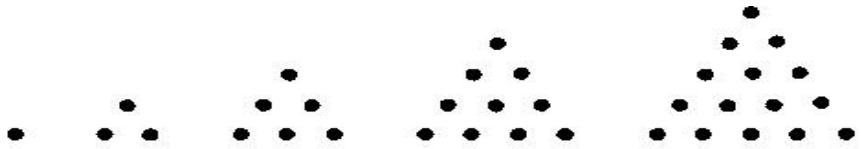
7. Find the nth term of the sequence $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{6}, \frac{1}{8}, -\frac{1}{10}, \ldots$

A) $a_n = \frac{(-1)^2}{2n}$
B) $a_n = \frac{(-1)^2}{2}$
C) $a_n = \frac{-1}{2n}$
D) $a_n = -\frac{1}{2n}$

8. What are the next four terms of the sequence 2, 3, 5, 8, ….?

A) 12, 17, 22, 27
B) 12, 20, 33, 54
C) 13, 21, 34, 55
D) 13, 18, 23, 28

9. Consider the figures below, what is the nth term of the sequence?



A) $\frac{n+1}{n}$ B) $\frac{n(n+1)}{2}$ C) $\frac{2n}{n+1}$ D) $\frac{n^2+1}{2}$

10. Which numerical pattern follows the rule "subtract 2, then multiply by 3", when starting with 5?

A) 5, 7, 21, 69
B) 5, 3, 6, 4, 12
C) 5, 9, 14, 36
D) 5, 3, 9, 7, 21

11. Which is the next ordered pair in the pattern (1, 6), (3, 18), (5, 30)?

A) (7, 49) B) (7, 42) C) (8, 56) D) (8, 64)

12. In the sequence, $a_n = \frac{(-1)^{n+1}(n-1)(n+2)}{n}$, what is a_5?

A) $\frac{28}{5}$ B) $-\frac{24}{5}$ C) $\frac{30}{5}$ D) $-\frac{26}{5}$

13. What is the rule for this pattern? 1st term:32, 2nd term:36, 3rd term:40

A) $4x$ B) $x + 4$ C) $4x + 28$ D) $4x + 32$

14. Which of the following patterns shows infinite sequence?

A) $6, 12, 18, 24, 30$
B) English Alphabets
C) First 20 whole numbers
D) $100, 50, 25, 12.5,\ldots$

15. Madison has the number pattern $\{5, 3, 8, 6, 11\}$ for a homework problem. She says that the number pattern adds 5 to the first number and then subtracts 2 to get the next number. Did Madison correctly describe the pattern?

A) No, the pattern adds 5 to the first number and then subtracts 2.
B) No, the number pattern subtracts 2 to the first number and then adds 5.
C) No, the pattern subtracts adds 3 to the first number and then adds 4.
D) Yes, Madison's description of the pattern is correct.

Lesson 1 Generating Patterns

WHAT'S IN

When you were in grade 8, you learned about concepts related to generating patterns like Inductive Reasoning. The knowledge and skills you acquired are very important for you to understand how to generate patterns and sequences. Hence, let us review inductive reasoning and perform the activities that follow.

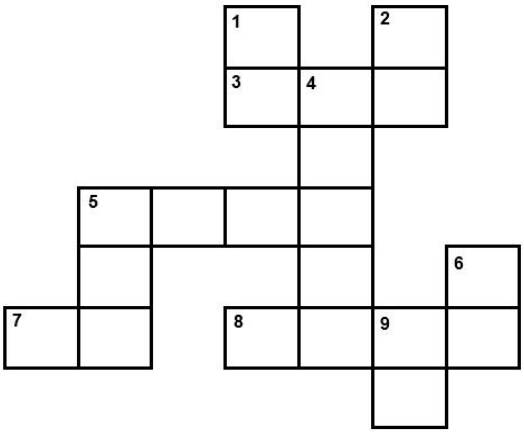
review

Inductive Reasoning is the process of observing data, recognizing patterns, and making generalizations (conjecture) from observations. A **conjecture** is a conclusion made from observing data or an educated guess based on patterns.

Activity 1. Complete Me!

Direction: Make a conjecture about each pattern. Then use your conjecture to draw or write the next term in the pattern.

A) Complete the puzzle below by providing the needed term/s in each pattern.



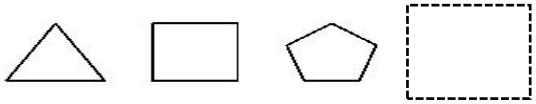
ACROSS

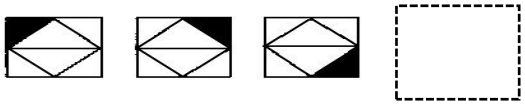
3) 1, 8, 27, 64, 125, ___
5) 4, 20, 100, 500, ___
7) 1, 1, 2, 3, 5, 8, ___
8) J, F, M, A, M, J, J, A, __, __, __, __

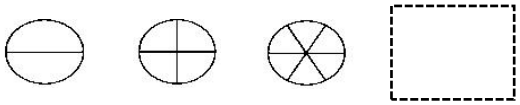
DOWN

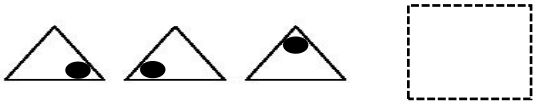
1) 2, 4, 6, 8 10, ___
2) 128, 64, 32, ___
4) 1, 10, 100, 1000, ___
5) 3, 9, 27, 81, ___
6) 1A, 2B, 3C, ____
9) O, T, T, F, F, S, S, E, ___, ___

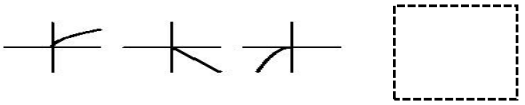
B) Draw the next object in each picture pattern.

1) 

2) 

3) 

4) 

5) 

6) 

What's New

At this point of the module, you are about to learn the Introduction to Sequences and Patterns. To understand better how to generate patterns, you will have to perform the simple activity below.

Activity 2. Let's Discover!

Direction: Read, understand, and perform the given instructions below then answer the questions asked.

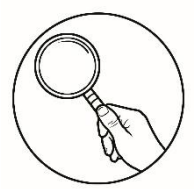
Materials: string, pair of scissors

1) Prepare five (5) strings with equal lengths.
2) Cut the first string once. (a)How many pieces are there? _____
   Cut the second string twice. (b) How many pieces are there?_____
   Cut the third string thrice. (c) How many pieces are there?_____
   Cut the fourth string four times. (d) How many pieces are there?_____
   Cut the fifth string five times. (e) How many pieces are there?_____
3) Based from your answers, complete the table below.

Number of cuts (x)	1	2	3	4	5
Number of pieces (*y*)					

4) Without cutting a string 6 times, how many pieces are there? _____
5) Have you seen a pattern? If yes, describe the pattern and state your conjecture. Use a formula or equation in your conjecture, where ***y*** is the number of pieces and ***x*** is the number of cuts.
6) Using your conjecture, how many pieces of strings can be made from (a)12 cuts? (b) 24 cuts? (c) 35 cuts? and (d) 42 cuts? *Show your solutions.*

Were you able to complete the task? If yes, you may proceed to the next page. If no, take time to finish for you to better understand the next discussions.



What Is It

How did you find activity 2? Have you given idea on how to generate a pattern? Let us process your answers.

1. Based from the task, the complete solution is shown in the table below:

Number of cuts (x)	1	2	3	4	5	6
Number of pieces (y)	**2**	**3**	**4**	**5**	**6**	**7**

2. From the table, notice that the number of pieces (y) of strings is one more than the number of cuts (x). Thus, we can state our conjecture as, "The number of pieces (**y**) when a string is cut **x** times can be computed using the formula **y = x + 1**."

3. Using the formula **y = x + 1,** we can now solve the number of pieces of strings that can be made from 12 cuts? 24 cuts? 35 cuts? and 42 cuts?

   a) 12 cuts, $x = 12$ y = 12 + 1 = **13**
   b) 24 cuts, $x = 24$ y = 24 + 1 = **25**
   c) 35 cuts, $x = 35$ y = 35 + 1 = **36**
   d) 42 cuts, $x = 42$ y = 42 + 1 = **43**

Were you able to get the same answers? If yes, very good! If no, I hope you were able to understand the discussions above. Based from the given activity, the number of pieces, $y = x + 1$, when a string is cut x times represents a sequence. Thus, the values of y which are $2, 3, 4, 5, 6, 7,\ldots$ is an example of a sequence.

The word ***sequence*** means an order in which one thing follows another in succession. A sequence is an ordered list. For another example, if we write x, $2x^2$, $3x^3$, $4x^4$, $5x^5$, ?, what would the next term in the sequence be—the one where the question mark now stands? The answer is $6x^6$.

definition

A ***sequence*** is a set of objects which is listed in a specific order, one after another. Each member or element in the sequence is called **term**. The terms in a sequence can be written as $\boldsymbol{a_1, a_2, a_3, a_4,\ldots, a_n,\ldots}$ which means a_1 is the first term, a_2 is the second term, a_3 is the third term, …, a_n is the nth term, and so on.

Sequences are classified as finite and infinite. A **finite** sequence contains a limited number of terms. This means it has an end or last term. Consider the examples below.

a) Days of the week: $\{Sunday, Monday, Tuesday,...,Saturday\}$
b) First 10 positive perfect squares: $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

On the other hand, an **infinite** sequence contains a countless number of terms. The number of terms of the sequence continues without stopping or it has no end term. The ellipsis (**...**) at the end of the following examples shows that the sequences are infinite. Consider the examples below.

a) Counting numbers: $\{1, 2, 3, 4, 5,...\}$
b) Multiples of 5: $\{5, 10, 15, 20, 15,...\}$

Sometimes a pattern in the sequence can be obtained and the sequence can be written using a ***general term***. In the previous example $x, 2x^2, 3x^3, 4x^4, 5x^5, 6x^6,...,$ each term has the same exponent and coefficient. We can write this sequence as $a_n = nx^n$, $n = 1, 2, 3, 4, 5, 6,...$, where $\boldsymbol{a_n}$ is called the ***general*** or ***nth term***.

A. Finding several terms of a sequence, given the general term:

Example 1.

Find the first four terms of the sequence $a_n = 2n - 1$.

Solution: To find the first term, let $n = 1$

$a_n = 2n - 1$	use the given general term
$a_{\mathbf{1}} = 2(\mathbf{1}) - 1$	substitute n by 1
$a_1 = 2 - 1$	perform the operations
$a_1 = 1$	simplify

Repeat the same process for the second to the fourth terms.

Find the second term, $n = 2$ $a_2 = 2(2) - 1 = 4 - 1 = 3$

Find the third term, $n = 3$ $a_3 = 2(3) - 1 = 6 - 1 = 5$

Find the fourth term, $n = 4$ $a_4 = 2(4) - 1 = 8 - 1 = 7$

Therefore, the first four terms of the sequence are **1, 3, 5, 7.**

Example 2.

Find the 5th to the 8th terms of the sequence $b_n = \frac{(-1)^n}{n+1}$.

Solution: To find the 5th term, let $n = 5$

$b_n = \frac{(-1)^n}{n+1}$ use the given general term

$b_5 = \frac{(-1)^5}{5+1}$ substitute n by 5

$b_5 = \frac{-1}{6} = -\frac{1}{6}$ simplify (– 1 raised to an odd number power is always negative)

Repeat the same process for the 6th to the 8th terms.

Find the 6th term, $n = 6$ $b_6 = \frac{(-1)^6}{6+1} = \frac{1}{7}$

Find the 7th term, $n = 7$ $b_7 = \frac{(-1)^7}{7+1} = \frac{-1}{8} = -\frac{1}{8}$

Find the 8th term, $n = 8$ $b_8 = \frac{(-1)^8}{8+1} = \frac{1}{9}$

Therefore, the 5th to the 8th terms of the sequence are $-\frac{1}{6}, \frac{1}{7}, -\frac{1}{8}, \frac{1}{9}$.

B. Finding the general term, given several terms of the sequence:

Example 3.

Write the general term of the sequence 5, 12, 19, 26, 33,...

Solution: Notice that each term is 7 more than the previous term. We can search the pattern using a tabular form.

Term	Given	Pattern	
1	5	5	5 + 7(0)
2	12	5 + 7	5 + 7(1)
3	19	5 + 7 + 7	5 + 7(2)
4	26	5 + 7 + 7 + 7	5 + 7(3)
5	33	5 + 7 + 7 + 7 + 7	5 + 7(4)
n	**a_n**	5 + 7 + 7 + 7 + 7 +...+ 7	**5 + 7(n – 1)**

In the pattern, the number of times that 7 is added to 5 is one less than the nth term (n – 1). Thus,

$a_n = 5 + 7(n - 1)$ equate a_n and $5 + 7(n - 1)$

$a_n = 5 + 7n - 7$ apply distributive property of multiplication

$a_n = 7n - 2$ combine similar terms

Therefore, the nth term of the sequence is $\mathbf{a_n = 7n - 2}$, where $n = 1, 2, 3, 4, 5,...$

Example 4.

Write the general term of the sequence $2, 4, 8, 16, 32, \ldots$

Solution: Notice that each term is 2 times the previous term. We can search the pattern using a tabular form.

Term	Given	Pattern	
1	2	2	2^1
2	4	2(2)	2^2
3	8	2(2)(2)	2^3
4	16	2(2)(2)(2)	2^4
5	32	2(2)(2)(2)(2)	2^5
n	$\mathbf{a_n}$	2(2)(2)(2)(2)…(2)	$\mathbf{2^n}$

Therefore, the nth term of the sequence is $\boldsymbol{a_n = 2^n}$, where $n = 1, 2, 3, 4, 5,\ldots$

Example 5.

Find the general term of the sequence $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25},\ldots$

Solution: $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25},\ldots$ write 1 as $\frac{1}{1}$

$\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}, \frac{1}{5^2},\ldots,\frac{1}{n^2}$ notice each denominator is an integer squared

Therefore, the nth term of the sequence is $\boldsymbol{a_n = \frac{1}{n^2}}$, where $n = 1, 2, 3, 4, 5,\ldots$

What's More

Now, it's your turn to apply the concepts on sequences and patterns to find the specified terms of a sequence when given its general term and vice versa.

Activity 3. Your Turn!

Direction**:** Answer what is asked in each set of exercises on a separate sheet of paper.

- In Exercises 1 – 4, write the first four terms of each sequence. Assume n starts at 1.

  1) $a_n = n$

  2) $a_n = \frac{n}{n+1}$

  3) $a_n = (-1)^{n+1}n^2$

  4) $a_n = \frac{n(n+1)}{2}$

- In Exercises 5 – 8, find the indicated term of each sequence given.

  5) $a_n = \left(\frac{1}{2}\right)^n \quad a_9 = \ ?$

  6) $a_n = \frac{(n+1)^2}{n-9} \quad a_{14} = \ ?$

  7) $a_n = \frac{(-1)^{n+1}(n-1)(n+2)}{n} \quad a_7 = \ ?$

  8) $a_n = \left(\frac{n}{9} - 12\right)^n \quad a_{99} = \ ?$

- In Exercises 9 – 12, write an expression for the nth term of the given sequence. Assume n starts at 1.

  9) $2, 4, 6, 8, 10,\ldots$

  10) $1, -1, 1, -1, 1,\ldots$

  11) $3, 9, 27, 81,\ldots$

  12) $\frac{1}{2\cdot1}, \frac{1}{3\cdot2}, \frac{1}{4\cdot3}, \frac{1}{5\cdot4}, \frac{1}{6\cdot5},\ldots$

What I Have Learned

Great! You have reached this part of the module. To ensure your full understanding on the concepts related to generating patterns, it's important that you are able to complete each statement below.

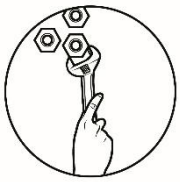
A sequence is _____

A term is _____

A finite sequence is _____ while infinite sequence is _____

To find the specified term/s of a sequence when given the general term, _____

To write the general term of a sequence when given some terms, _____



What I Can Do

Generating patterns is a vital concept in performing any mathematical investigation. Similar to the previous activity on the number of pieces when a string is cut **x** times, a sequence is formed when a repeated process following a certain rule is employed. To perform a simple mathematical investigation, perform the task below.

Activity 4. Let's Investigate!

Direction: Given the figures below, perform a mathematical investigation by following the given steps.

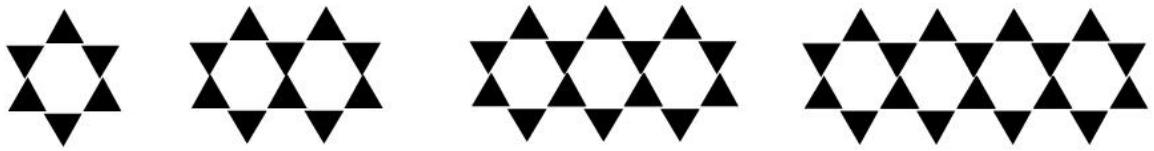


Figure 1 Figure 2 Figure 3 Figure 4

Step 1. To investigate, you are going to make your own problem to solve. Based from the figures above, what do you think could be a probable problem to investigate?

Step 2. Identify your variables (dependent and independent variables). For example, number of pieces of strings, a_n, and number of cuts n. Then list your data.

Step 3. Present your data in a tabular form.

Term	Sequence	Pattern	
1			
2			
3			
4			
n	a_n		

Step 4. State your conjecture using a general or n^{th} term.

Step 5. Test your conjecture. Using your derived formula or general term, find the first four terms of your sequence if it matches.

Assessment

Let us determine how much you have learned from this module. Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1. Which of the following shows a pattern?
   A) 3, 2, 3, 2, 3, 2
   B) 5, 7, 2, T, 6, Y
   C) A, G, M, T, 0, 9
   D) smooth, long, wall, sing

2. Which of the following defines infinite sequence?
   A) days of the week
   B) teenage life
   C) every other day
   D) first Fridays of July 2020

3. What are the next three terms of the sequence 1, 11, 22, 34, ….?
   A) 46, 60, 75 B) 47, 61, 76 C) 42, 54, 66 D) 44, 60, 74

4. What is the 25th term of the sequence $a_n = \frac{(-1)^n}{n^2}$?
   A) $\frac{1}{625}$ B) $-\frac{1}{625}$ C) $\frac{25}{625}$ D) $-\frac{25}{625}$

5. What is the 11th term in the sequence $-1, 4, -9, 16, -25, ..$?
   A) 100 B) -100 C) 121 D) -121

6. Find the first four terms of the sequence $a_n = 3n + 2$.
   A) 5, 7, 11, 14 B) 5, 8, 11, 15 C) 5, 8, 11, 14 D) 5, 9, 13, 17

7. Which numerical pattern follows the rule “subtract 2, then multiply by 3”, when starting with 5?
   A) 5, 7, 21, 69 B) 5, 3, 9, 7, 21 C) 5, 3, 6, 4, 12 D) 5, 9, 14, 36

8. What rule will correctly describe the sequence: 2, 6, 12, 20, 30,…?
   A) $n + 1$ B) $n^2 + 1$ C) $2n + 1$ D) $n^2 + n$

9. Find the nth term of the sequence $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{6}, \frac{1}{8}, -\frac{1}{10}, \ldots$
   A) $a_n = \frac{-1}{2n}$ B) $a_n = \frac{(-1)^2}{2}$ C) $a_n = \frac{(-1)^2}{2n}$ D) $a_n = -\frac{1}{2n}$

10. Which is the next ordered pair in the pattern $(2, 1), (4, 4), (6, 9)$?

A) $(8, 12)$ B) $(7, 16)$ C) $(8, 16)$ D) $(7, 12)$

11. What is the 8^{th} term in the sequence 9, 4, -1, -6, -11, …?

A) -21 B) -26 C) -31 D) -36

12. In the sequence, $a_n = \frac{(-1)^{n+1}(n-1)(n+2)}{n}$, what is a_{10}?

A) $\frac{108}{10}$ B) $-\frac{54}{5}$ C) $\frac{52}{5}$ D) $-\frac{104}{10}$

13. Which of the following patterns shows finite sequence?

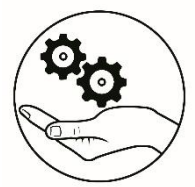
A) 6, 12, 18, 24, 30,...
B) multiples of 6
C) First 20 whole numbers
D) 100, 50, 25, 12.5,...

14. Find the general term of the sequence $3, 9, 27, 81,...$

A) $3n$ B) n^3 C) 3^n D) $n + 3$

15. Write the first four terms of the sequence $a_n = n^2 - 1$.

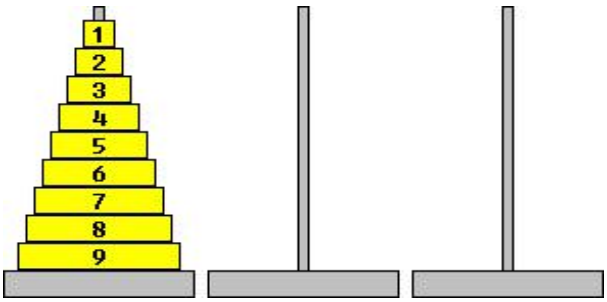
A) 0, 3, 8, 15 B) 1, 3, 5, 7 C) 1, 5, 10, 16 D) 0, 2, 7, 12



Additional Activities

Awesome! Before we end this module, let me introduce a puzzle game called Tower of Hanoi. Are you familiar with this game? If not, allow me to introduce it to you.

In the **Tower of Hanoi** puzzle a player attempts to move a large pile of disks, known as the **Tower**, from the leftmost peg to the rightmost on the puzzle board. The rules of the puzzle state that the player can only move one disk per turn and can never place a larger disk onto a smaller one at any time.



If you are interested to play the puzzle, you can search it on the internet. For the meantime, I just want to use the puzzle game for the purpose of applying sequences and generating patterns.

Situation: In playing the game, you can choose the number of disks of your tower and play with the least possible moves. The least number of moves when playing the puzzle with respect to the number of disks are as follows:

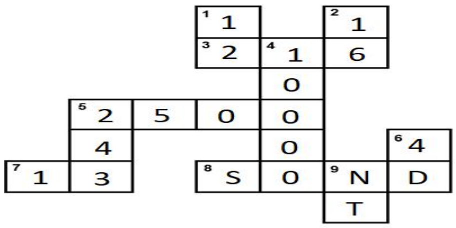
Number of Disks	**Number of Moves**	*Number of Disks*	**Number of Moves**
1	**1**	*5*	**31**
2	**3**	*6*	**63**
3	**7**	*7*	**127**
4	**15**	*9*	**255**

Task: Based on the above data, find the general term in finding the number of moves with respect to the number of disks in playing the tower of Hanoi.

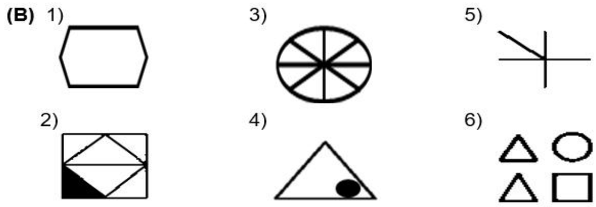
KEY ANSWERS

What I Know	(1) C	(4) C	(7) A	(10) D	(13) B
	(2) D	(5) A	(8) C	(11) B	(14) D
	(3) B	(6) D	(9) B	(12) A	(15) C

What's In (Activity 1)



(B)



What's More (Activity 3)

1) $1, 2, 3, 4$	4) $1, 3, 6, 10$	7) $\frac{54}{7}$	10) $a_n = (-1)^{n+1}$
2) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$	5) $\frac{1}{512}$	8) -1	11) $a_n = 3^n$
3) $1, -4, 9, -16$	6) 45	9) $a_n = 2n$	12) $a_n = \frac{1}{(n+1)\cdot n}$

What Can I Do (Activity 4)

No. of Hexagons (n)	No. of Triangles (a_n)	Pattern	
1	6	6	6 + 4(0)
2	10	6 + 4	6 + 4(1)
3	14	6 + 4 + 4	6 + 4(2)
4	18	6 + 4 + 4 +4	6 + 4(3)
N	a_n	6 + 4 + 4 + 4 + … + 4	**6 + 4(n-1) or 4n + 2**

Conjecture:
The total number of triangles, a_n, used in the n[th] term hexagonal sequence is $\mathbf{4n+2}$.

Assessment	(1) A	(4) B	(7) B	(10) C	(13) C
	(2) C	(5) D	(8) D	(11) B	(14) C
	(3) B	(6) C	(9) C	(12) B	(15) A

Additional Activities
The least possible number of moves, a_n, with respect to the number of disks (n) can be computed using the general term $\boldsymbol{a_n = 2^n - 1}$.

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For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph