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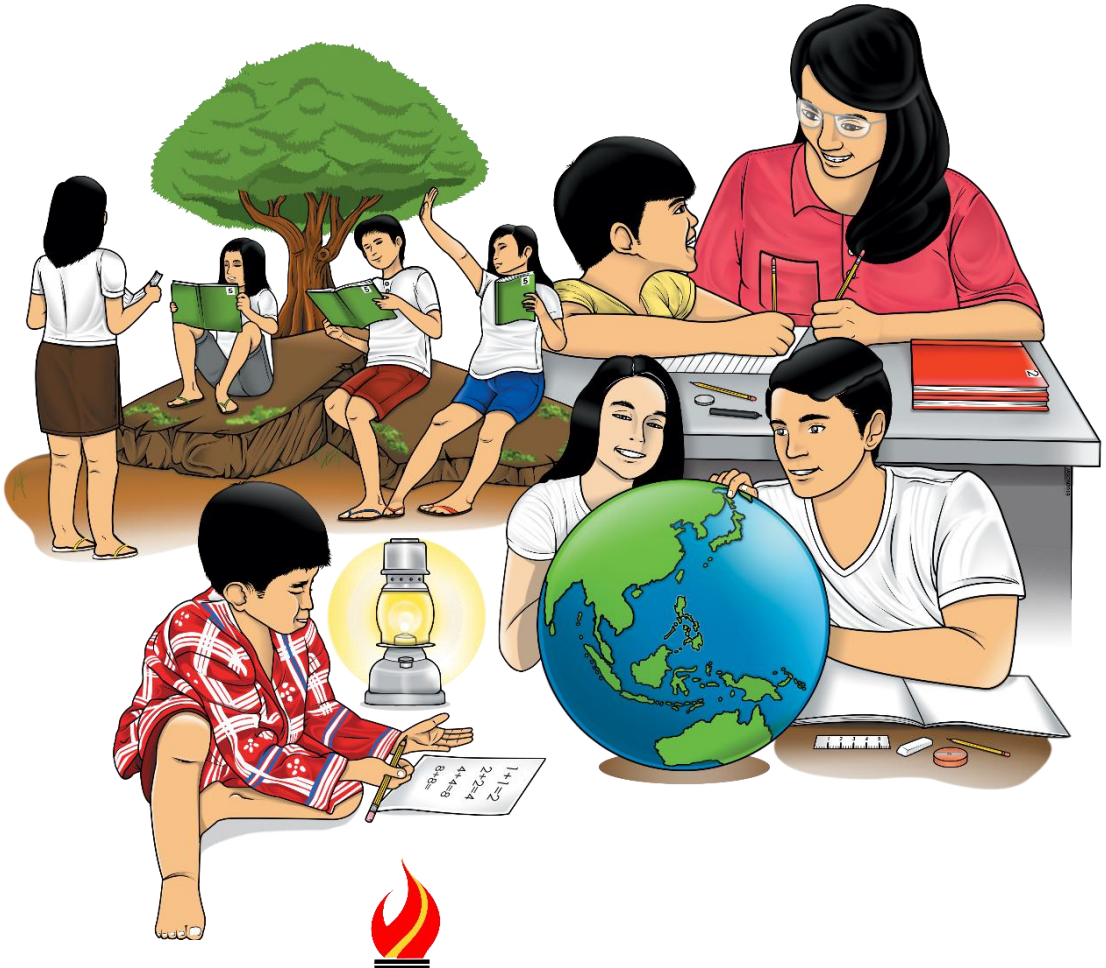
DEPARTMENT OF EDUCATION

Mathematics

Quarter 1 – Module 7
Geometric Series



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Introductory Message

This is the seventh learning competency in our Mathematics 10 curriculum standards hence mastery of the skills is significant to have a smooth progress in the succeeding lessons.

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling

For the facilitator:

Being the facilitator of this module, please be patient in orienting the learner about the importance of this module. Kindly emphasize to the learner that this module is a big contribution. By the way, do not forget to remind the learner to use separate sheets in answering all of the activities found in this module.

For the learner:

Hello learner. I hope you are ready to progress in your Grade 10 Mathematics by accomplishing this learning module. This is designed to provide you with interactive tasks to further develop the desired learning competencies prescribed in the K-12 Curriculum of the Department of Education. In this module, you are expected to find the sum of the terms of a given finite or infinite geometric series.

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
	**What I need to know**	This will give you an idea of the skills or competencies you are expected to learn in the module.
	**What I know**	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.
	**What's In**	This is a brief drill or review to help you link the current lesson with the previous one.
	**What's New**	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.

	**What Is It**	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	**What's More**	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	**What I have Learned**	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
	**What I Can Do**	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
	**Assessment**	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	**Additional Activities**	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned
	**Answer Key**	This contains answers to all activities in the module.

At the end of this module you will also find:

References This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

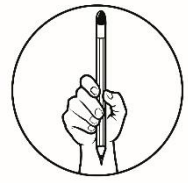
We hope that though this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module aims to provide the material necessary to introduce the mathematical concept of geometric sequences to Grade – 10 students. This module will discuss the formula in finding the sum of finite and infinite geometric series. It also includes interesting activities which will help learners understand well geometric series.

After going through this module, the learner should be able to:

a. find the sum of terms of a finite geometric sequence, and
b. find the sum of terms of infinite geometric sequence.



What I Know

A. Multiple Choice. Read and analyze the following items and determine the letter of the correct answer from the given choices. Write your answer on a separate sheet of paper.

_____ 1. This refers to the sum of the terms of a geometric sequence.

A. Series
B. Limit
C. Continuity
D. All of these

_____ 2. Find the sum of the first six terms of a geometric sequence whose first term is 2 and common ratio of $\frac{2}{3}$.

A. $\frac{1\,234}{4\,567}$
B. $\frac{1\,330}{243}$
C. $\frac{3\,990}{729}$
D. $\frac{760}{4\,551}$

_____ 3. Find the sum of the first five terms of the geometric sequence 4, 6, 9, …

A. $\frac{11}{4}$
B. $\frac{211}{4}$
C. $\frac{4}{11}$
D. $\frac{4}{211}$

_____ 4. Find the sum of the first five terms of the geometric sequence 2, 8, 32, 128, …

A. 243
B. 682
C. 679
D. 743

_____ 5. Find the sum of the terms of the infinite geometric sequence 45, 15, 5, …

A. 68
C. $\frac{135}{2}$
B. $\frac{137}{2}$
D. 69

For items 6 – 10, find specified geometric series of the following geometric sequences:

_____ 6.) 3, 12, 48, … $\boldsymbol{S_7}$
_____ 7.) 2, 6, 18, … $\boldsymbol{S_6}$
_____ 8.) 125, 25, 5,… $\boldsymbol{S_8}$
_____ 9.) First term $a_1 = 2$ and common ratio $r = -4$; find $\boldsymbol{S_8}$
_____ 10.) First term $a_1 = \frac{1}{2}$ and common ratio $r = -2$; find $\boldsymbol{S_6}$

For items 11 – 15, find the geometric series of each infinite geometric sequence.

_____ 11.) 12, 6, 3, …
_____ 12.) 125, 25, 5, …
_____ 13.) 1, $\frac{1}{5}$, $\frac{1}{25}$, …
_____ 14.) 15, 5, $\frac{5}{3}$, …
_____ 15.) 1, $\frac{2}{3}$, $\frac{4}{9}$, …

Lesson

1 Finite Geometric Series

What's In

In the previous module, we derived the formula in finding the *nth* term of a geometric sequence. This formula allows you to accurately identify the *nth* term of any geometric sequence.

We shall now proceed on discussing how to find the sum of the terms of geometric sequences.

Consider the geometric sequence 3, 6, 12, 24, … If I let you find the sum of the first five terms of the geometric sequence, maybe, you'll simply generate the five terms then add them. For example, **3 + 6 + 12 + 24 + 48 = 93**.

Essential Question:

What is the sum of the first 10 terms? First 15 terms? First 20 terms?

How will you be able to answer the above question? Will you do the same of adding the terms one by one to find the sum? How long will it take you to answer the question?

This module will discuss to you on how to find the sum of the terms of finite and infinite geometric sequence without going through the process of adding the terms one by one.

What's New

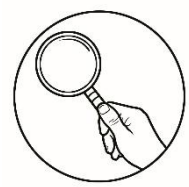
PLUS FACTOR

Find the sum of the first five terms of the following geometric sequences.

1.) 7, 14, 28,...
2.) 3, 12, 48,...
3.) 100, 50, 25,...
4.) 1, 6, 36,...
5.) 54, 18, 6,...

Basically, you will identify first the common ratio of the sequence to generate the next terms of the sequence. For Item number 1, the common ratio (r) is equal to $\frac{14}{7}$= 2. Therefore, the first five terms of the sequence are 7, 14, 28, 56, and 112. Then, add the terms to find the sum.

$$\mathbf{7 + 14 + 28 + 56 + 112 = 217}$$



What Is It

Consider item number 1 on your activity, the sum of the first five terms obtained is 217. You were able to find the sum by generating all the terms and then adding them. But, how about the sum of the first 20 terms? Are you still going to generate all the terms?

7 + 14 + 28 + 56 + 112 → *Geometric Series*

Geometric Series refers to the sum of the terms of a geometric sequence.

Here is the formula in finding the sum of the first n terms of a finite geometric series:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Where: S_n= the sum of the first n terms
a_1= first term
n = number of terms
r = common ratio

Let's try to use the formula.

Find the sum of the first five terms of the geometric series $7, 14, 28, \ldots$

Solution: $a_1 = 7$, $n = 5$, $r = \frac{a_2}{a_1} = \frac{14}{7} = 2$

Let's substitute the given values in the formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

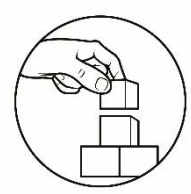
$$S_5 = \frac{7(1 - 2^5)}{1 - 2}$$

$$S_5 = \frac{7(1 - 32)}{-1}$$

$$S_5 = \frac{7(-31)}{-1}$$

$$S_5 = 217$$

Using the formula, it gives the same answer.



What's More

ACTIVITY 1. SUM IT UP

1. Find the sum of the first seven terms of a geometric sequence whose first term is 3 and common ratio is 4.

Solution : $a_1 = 3$, $n = 7$, $r = 4$

Substitute these values in the formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_7 = \frac{3(1 - 4^7)}{1 - 4}$$

$$S_7 = \frac{3(1 - 16,384)}{-3}$$

$$S_7 = \frac{7(-16,383)}{-3}$$

$$S_7 = \frac{-114,681}{-3}$$

$$S_7 = 38,227$$

2. Find the sum of the first 6 terms of geometric series $\frac{1}{4}, \frac{1}{2}, 1, \ldots$

Solution: $\boldsymbol{a_1 = \frac{1}{4}}$, $\boldsymbol{n = 6}$, $\boldsymbol{r = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{4}{2} = 2}$

Substitute these values in the formula:

$$\boldsymbol{S_n = \frac{a_1(1 - r^n)}{1 - r}}$$

$$\boldsymbol{S_6 = \frac{\frac{1}{4}(1 - 2^6)}{1 - 2}}$$

$$\boldsymbol{S_6 = \frac{\frac{1}{4}(1 - 64)}{-1}}$$

$$\boldsymbol{S_6 = \frac{\frac{1}{4}(-63)}{-1}}$$

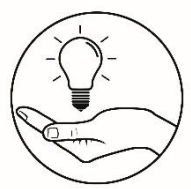
$$\boldsymbol{S_6 = \frac{-\frac{63}{4}}{-1}}$$

$$\boldsymbol{S_6 = \frac{63}{4}}$$

ASSESSMENT 1. PLUS IT!

1. Find the sum of the first 6 terms of a geometric sequence whose first term is 2 and common ratio is $\frac{2}{3}$.

2. What is the sum of the first 10 terms of the geometric sequence 4, 2, 1?



What I Have Learned

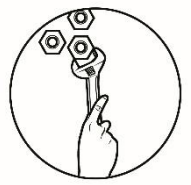
COMPLETE ME!

Fill in the blanks.

1. _____ is the sum of the terms of a geometric sequence.
2. To find finite geometric series, use the formula _____.

For numbers 3 – 5, use the sequence 3,6,12,... Determine

3. a_1 = _____
4. r = _____
5. S_5 = _____



What I Can Do

Illustrative Example:

Joey saves an amount in his bamboo bank each week. To make it fun, he doubles whatever amount is inside the bank next week. On the first week, he saves 1 peso. On the 10th week, how much will be in the bamboo bank in all?

	First Week	Second Week	Third Week	Fourth Week
Amount	₱1	₱2	₱4	₱8

The total savings can be computed by

$$1 + 2 + 4 + 8 + \ldots + 512 \longrightarrow \boldsymbol{S_{10}}$$

$$\boldsymbol{a_1 = 1, r = 2, S_{10} = ?}$$

$$\boldsymbol{S_n = \frac{a_1(1 - r^n)}{1 - r}}$$

$$\boldsymbol{S_{10} = \frac{1(1 - 2^{10})}{1 - 2}}$$

$$\boldsymbol{S_{10} = 1{,}023}$$

Therefore, Joey has a total savings of **₱1,023** after ten weeks.

Problem: Solve completely.

Suppose Rico saves P100.00 in January and each month thereafter he manages to save one-half more than what he saved in the previous month. How much is Rico's savings after 10 months? Round off your final answer to two decimal places.

Lesson 2 Infinite Geometric Series

What's In

In the previous lesson, you have learned about finding the sum of the terms of a finite geometric series using a formula. Since the sequence is finite, we can easily find the sum. But, what if the sequence has no last term? Can you possibly find the sum?

This module will discuss to you how to find the sum of terms of infinite geometric sequences.

What's New

INFINI-TERM

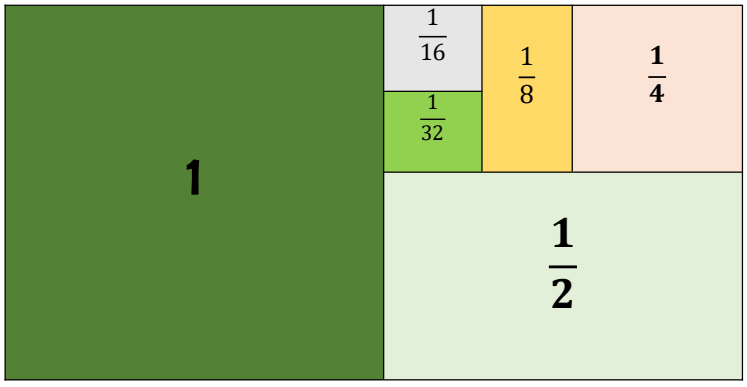
Task: Find the sum of the terms of the geometric sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

You might be wondering how you can get the sum of the terms of the geometric sequence because its terms are infinite. By the way, is it even possible?

If you want to know the answer to that question, you continue learning the next lesson.

What Is It

Consider a 1 by 2 rectangular figure. Divide the figure into various portions as shown below.



The rectangle was divided into various portions such as 1 unit, $\frac{1}{2}$unit, $\frac{1}{4}$ unit, $\frac{1}{8}$ unit, $\frac{1}{16}$unit and so on. The rectangle can still be divided into smaller rectangles up to infinity.

If the areas of all rectangles are added this will give a sum of 2 square units. This scenario indicates that you can still find the sum of the terms of an infinite geometric sequence.

In a mathematical statement, we can write the sum of the areas of the rectangles as follows:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \ldots = 2$$

Sum of terms of an infinite geometric sequence

Ellipsis is a set of three dots that indicates infinity.

Based from this example, we can have the formula:

$$S_\infty = \frac{a_1}{1 - r}$$

Where: S_∞ =sum to infinity
a_1 = first term
r = common ratio

Let us test if the formula is correct:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots = 2$$

Solution:

$$a_1 = 1 \qquad r = \frac{a_2}{a_1} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

Let us substitute these values in the formula:

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{1}{1-\frac{1}{2}}$$

$$S_\infty = \frac{1}{\frac{1}{2}}$$

$S_\infty = 2$ ✓

We tested it right!

TAKE NOTE

This formula is applicable only to infinite geometric series when the common ratio is lesser than 1 but greater than -1.

$$-1 < r < 1$$

What's More

ACTIVITY 1. SUM UP TO INFINITY

1. Find the sum of the terms of the infinite geometric sequence 3, 1, $\frac{1}{3}$,...

   Solution: $a_1 = 3$, $r = \frac{1}{3}$

   Substitute these values in the formula:

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{3}{1-\frac{1}{3}}$$

$$S_\infty = \frac{3}{\frac{2}{3}}$$

$$S_\infty = \frac{9}{2}$$

2. Find the sum of an infinite geometric sequence whose first term is 5 and common ratio is $\frac{1}{4}$.

Solution : $a_1 = 5$, $r = \frac{1}{4}$

Substitute these values in the formula:

$$S_\infty = \frac{a_1}{1 - r}$$

$$S_\infty = \frac{5}{1 - \frac{1}{4}}$$

$$S_\infty = \frac{5}{\frac{3}{4}}$$

$$S_\infty = \frac{20}{3}$$

ASSESSMENT 1. INFINITY

Solve for the specified geometric series. Show your complete solutions.

1.) Find the sum of the terms of an infinite geometric sequence whose first term is 4 and common ratio is $\frac{1}{5}$.

2.) Given the sequence 9, 3, 1,... , find S_∞

What I Have Learned

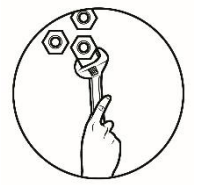
COMPLETE ME!

Fill in the blanks.

1. _____ is a set of three dots that indicates infinity.
2. To find infinite geometric series, use the formula _____.
3. In an infinite geometric series, $-1 <$ ___ > 1.

For numbers $4-5$, use the sequence $2,1,\frac{1}{2},\ldots$ Determine

4. $r =$ _____
5. $S_\infty =$ _____



What I Can Do

Illustrative Example:

A ball tossed to a height of 8 meters rebounds to $\frac{1}{2}$ its previous height. Find the distance the ball has travelled when it comes to rest.

Note: 1) The distance travelled going up is the same as the distance travelled going down.

2) When the ball is at rest, the distance is zero.

Direction	Tossing	1^{st} Rebound	2^{nd} Rebound	3^{rd} Rebound
Up	$8m$	$4m$	$2m$	$1m$
Down	$8m$	$4m$	$2m$	$1m$
Total	$2(8)\ or\ 16m$	$2(4)\ or\ 8m$	$2(2)\ or\ 4m$	$2(1)\ or\ 2m$

The total distance can be written as

$2(8) + 2(4) + 2(2) + 2(1) + \ldots$ or $2(8 + 4 + 2 + 1 + \ldots)$ $\quad \boldsymbol{2S_\infty}$

$$\boldsymbol{a_1 = 8, r = \frac{1}{2}, S_\infty = ?}$$

$$\boldsymbol{S_\infty = \frac{a_1}{1 - r}}$$

$$\boldsymbol{S_\infty = \frac{8}{1 - \frac{1}{2}}}$$

$$\boldsymbol{S_\infty = 16 \longrightarrow 2S_\infty = 32}$$

Therefore, the total distance the ball has travelled when it comes to rest is $32m$.

Problem: Solve completely.

A ball tossed to a height of 6 meters rebounds to $\frac{2}{3}$ its previous height. Find the distance the ball has travelled when it comes to rest.

Assessment

A. Multiple Choice. Read and analyze the following items and determine the letter of the correct answer from the given choices. Write your answer on a separate sheet of paper. **USE CAPITAL LETTERS ONLY.**

_____ 1. How can you indicate infinite geometric sequences?

A. Use an ellipsis
B. Use an arrowhead
C. Use a bar line above the sequence
D. All of these

_____ 2. Find the sum of the first six terms of a geometric sequence whose first term is 2 and common ratio is $\frac{2}{3}$.

A. $\frac{1330}{243}$
B. $\frac{1234}{4567}$
C. $\frac{760}{4551}$
D. $\frac{3990}{729}$

_____ 3. Find the sum of the first five terms of the geometric sequence 4, 6, 9…

A. $\frac{11}{4}$ C. $\frac{4}{11}$

B. $\frac{4}{211}$ D. $\frac{211}{4}$

_____ 4. Find the sum of the terms of the infinite geometric sequence 45, 15, 5,…

A. 68 C. $\frac{135}{2}$

B. $\frac{137}{2}$ D. 69

_____ 5. Find the sum of the first five terms of the geometric series 2, 8, 32, 128,…

A. 682 C. 679

B. 243 D. 743

For items 6 – 10, find the specified geometric series of the following geometric sequences:

_____ 6) $3, 12, 48, \ldots$ $\boldsymbol{S_7}$

_____ 7) $2, 6, 18, \ldots$ $\boldsymbol{S_6}$

_____ 8) $125, 25, 5, \ldots$ $\boldsymbol{S_8}$

_____ 9) First term $a_1 = 2$ and common ratio $r = -4$; find $\boldsymbol{S_8}$

_____ 10) First term $a_1 = \frac{1}{2}$ and common ratio $r = -2$; find $\boldsymbol{S_6}$

For items 11 – 15, find the sum of the terms of each of the following infinite geometric sequences:

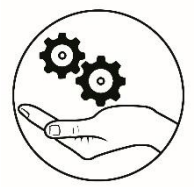
_____ 11) $12, 6, 3, \ldots$

_____ 12) $125, 25, 5, \ldots$

_____ 13) $1, \frac{1}{5}, \frac{1}{25}, \ldots$

_____ 14) $15, 5, \frac{5}{3}, \ldots$

_____ 15) $1, \frac{2}{3}, \frac{4}{9}, \ldots$



Additional Activity

Answer completely.

When a ball is tossed to a height of 4 meters above the ground, it always rebounds to 40% of its previous height until it stops. Find the total distance that the ball has covered when it strikes the ground for the fifth time.

Answer Keys

LESSON 1

What I Know

1. A
2. B
3. B
4. B
5. C
6. 16 383
7. 728
8. $\frac{97656}{625}$
9. – 26 214
10. $-\frac{21}{2}$
11. 24
12. $\frac{625}{4}$
13. $\frac{5}{4}$
14. $\frac{45}{2}$
15. 3

WHAT'S NEW

1. 217
2. 1 023
3. $\frac{775}{4}$
4. 1 555
5. $\frac{242}{3}$

What I Can Do

₱11,333.01

What I Have Learned

1. geometric series
2. $S_n = \frac{a_1(1-r^n)}{1-r}$
3. $a_1 = 3$
4. $r = 2$
5. $S_5 = 93$

Answer Keys

LESSON 2

What's More: Assessment 1

1. 5
2. $\frac{27}{2}$

What I Have Learned

1. Ellipsis
2. $S_\infty = \frac{a_1}{1-r}$
3. r
4. $\frac{1}{2}$
5. 4

ASSESSMENT

16. A
17. A
18. D
19. C
20. A
21. 16 383
22. 728
23. $\frac{1}{2,500}$
24. – 26 214
25. $-\frac{201}{2}$
26. 24
27. $\frac{625}{4}$
28. $\frac{5}{4}$
29. $\frac{45}{2}$
30. 3

What's More. Assessment 1

1. $\frac{1330}{243}$
2. $\frac{1023}{128}$

What I Can Do

$36m$

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