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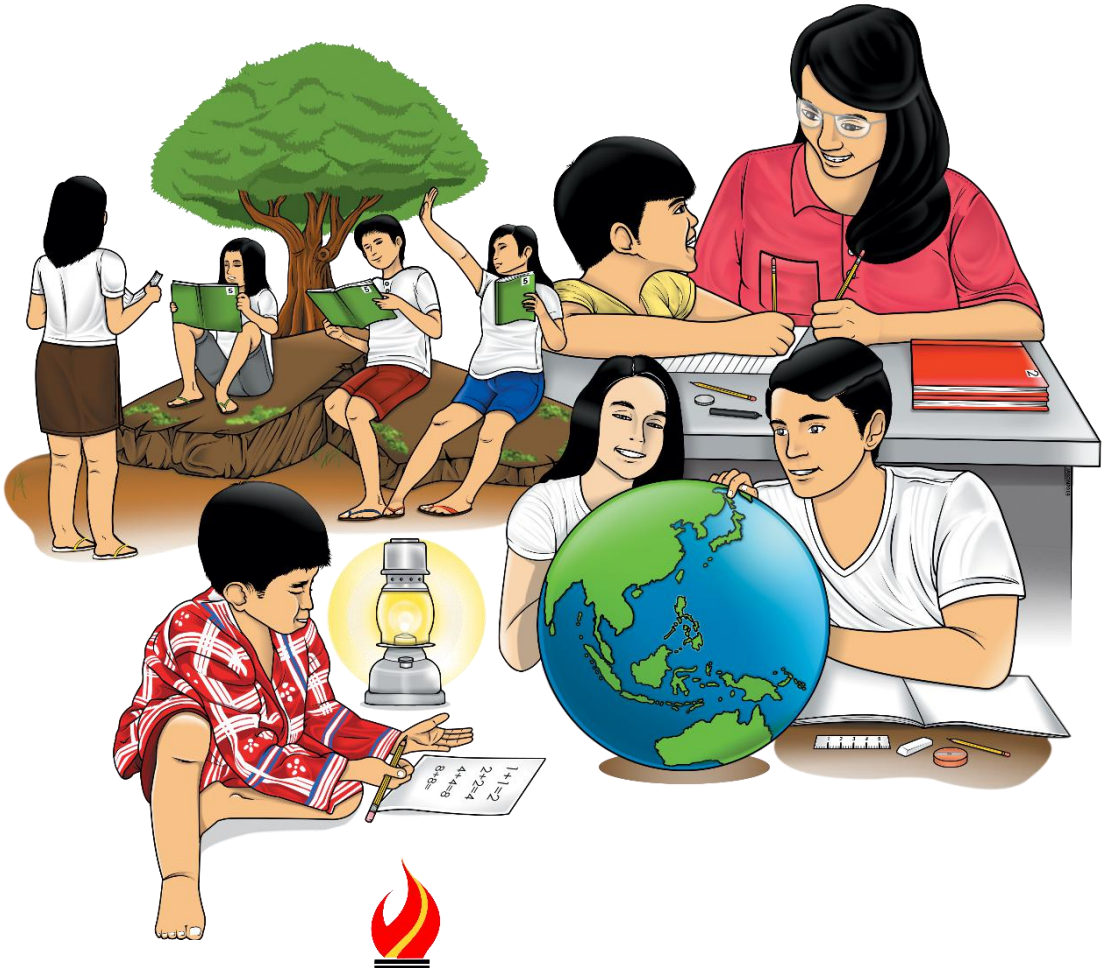
Mathematics

Quarter 1 – Module 9: Proving the Remainder and Factor Theorems



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Mathematics

Quarter 1 – Module 9:
Proving the remainder and factor theorems

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Introductory Message

This is the ninth learning competency in our Mathematics 10 curriculum standards hence mastery of the skills is significant to have a smooth progress in the succeeding lessons.

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

For the facilitator:

This module is intended to help grade 10 students understand and master the concepts of proving the remainder and factor theorems. This module also designed to equip the students with essential knowledge about Remainder Theorem and Factor Theorem. Please have patience in assisting the learners accomplish this module.

For the learner:

This module is the ninth learning competency in our Mathematics 10 curriculum standards hence mastery of the skills is significant for you to have a smooth progress in the succeeding lessons. This learning material serves as a bridge from synthetic division to remainder and factor theorems. By doing the prepared activities, it is expected from you to develop your skill in using the remainder theorem to understand the factor theorem. Please read completely the written texts and follow the instructions carefully so that you will be able to get the most of this learning material. We hope that you will enjoy learning!

Here is a guide on the parts of the learning modules which you need to understand as you progress in reading and analyzing its content.

ICON	LABEL	DETAIL
	**What I need to know**	This will give you an idea of the skills or competencies you are expected to learn in the module.
	**What I know**	This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.

	**What's In**	This is a brief drill or review to help you link the current lesson with the previous one.
	**What's New**	In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.
	**What Is It**	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	**What's More**	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
	**What I have Learned**	This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.
	**What I Can Do**	This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.
	**Assessment**	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	**Additional Activities**	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
	**Answer Key**	This contains answers to all activities in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that though this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

What I Need to Know

This module was designed and written with you in your mind. It is here to help you understand better on how to prove the remainder and factor theorems. The scope of this module permits it to be used in many different learning situations. The lessons are done to follow the standard sequence of the course.

LEARNING OBJECTIVES:

You should be able to:

1. Prove the remainder and factor theorems.
2. Find the remainder using synthetic division or the remainder theorem.
3. Solve word problems using the remainder and factor theorem.

What I Know

DIRECTION: Let us determine how much you already know about the remainder theorem and factor theorem. Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1) Which of the following binomials is a factor of the $P(x) = x^3 - 7x + 5$?

   a) $x - 1$ b) $x + 1$ c) $x + 2$ d) none of these

2) Is the first polynomial a factor of the second polynomial, $x - 1$; $x^2 + 2x + 5$?

   a) yes b) no c) uncertain d) invalid

3) Which is the missing factor in the equation $x^2 - 4 = (x - 2)$ (_____)?

   a) $x - 2$ b) $x + 2$ c) $x + 4$ d) $x - 4$

4) Which of the following is NOT a factor of $x^3 + 5x^2 - x - 5$?

   a) $x + 1$ b) $x - 1$ c) $x - 5$ d) $x + 5$

5) What are the factors of $x^2 - 2x - 24$?

a) $(x + 4)(x - 6)$ c) $(x - 12)(x + 1)$

b) $(x - 8)(x + 3)$ d) $(x + 12)(x - 12)$

6) What is the remainder when $x^3 - 4x^2 + x + 8$ is divided by $x - 2$?

a) 1 b) 2 c) 3 d) 4

7) When P(x) is divided by x – r and the remainder is equal to zero, it means _____.

a) x – r is a factor of P(x) c) P(x) is a factor of x – r

b) P(x) is a factor of x – r d) x – r is the only factor of P(x)

8) Evaluate the polynomial $x^3 + x^2 + x + 3$ if the value of x = -2.

a) 3 b) -3 c) 17 d) -17

9) Which polynomial gives a remainder of 0 when divided by 3x – 2?

a) $12x^2 + 15x - 18$ c) $12x^2 + 19x - 18$

b) $12x^2 + 18x + 7$ d) $12x^2 + 8x + 7$

10) Which of the following is a factor of $2x^2 - 5x + 3$?

a) x – 3 b) 2x + 3 c) x – 1 d) 3x + 2

11) What is the remainder if $x^2 - 7x - 4$ is divided by x – 2?

a) -14 b) 5 c) 6 d) 7

12) Which statement is true?

a) The quotient multiplied by the dividend plus the remainder equals the divisor.
b) If $x^2 + 5x + 7$ is divided by x + 2, the remainder is 1.
c) If the remainder is 0, then the dividend is a factor of the divisor.
d) The remainder is a factor of the dividend if the quotient is 0.

13) Which is the remainder if $2x^3 - 7x^2 - 19x + 20$ is divided by x – 5?

a) -1 b) 0 c) 1 d) 2

14) x – 3 is a factor of $x^4 - 3x^3 - x + 3$.

a) false b) true c) uncertain d) invalid

15) Find another factor of $x^3 - 7x^2 + 4x - 28$ if x – 7 is a factor.

a) $x^2 + x + 4$ b) $x^2 - 2x - 4$ c) $x^2 - x - 4$ d) $x^2 + 4$

Lesson 1 The Remainder Theorem and the Factor Theorem

What's In

In the earlier lesson, you have learned how to divide polynomials using long division or by synthetic division. Read and understand the discussion below, then investigate.

1. Divide the polynomial function $f(x) = x^3 - 2x^2 + x - 3$ by $x - 2$ using synthetic division.

2	1	-2	1	-3	*Solution:*
		2	0	2	
	1	0	1	-1	

2. Since the divisor is $x - 2$, evaluate the above function at $x = \mathbf{2}$.

Solution: $f(x) = x^3 - 2x^2 + x - 3$

$f(2) = 2^3 - 2(2^2) + 2 - 3$ Substitute x by 2

$f(2) = 8 - 8 + 2 - 3$ Simplify

$f(2) = -1$ Perform the operations

Guide Question: What can you say about the results of the two separate solutions?

Notice:

- When the function was divided by $x - 2$, the remainder is -1.
- When the function was evaluated at $x = 2$, the result is -1.
- This leads us to the ***Remainder Theorem***.

What's New

Definition

REMAINDER THEOREM
If a polynomial ***P(x)*** is divided by ***x – a***, then the remainder is

Example: If $f(x) = x^3 - 2x^2 + x - 3$ divided by x – 2, then the remainder is $r = P(a)$.

Based from the solutions above, the remainder r = -1, and f(2) = -1.
Thus, r = f(2).

When a polynomial is divided by x – a, if the remainder is zero, we say that x – a is a factor of the polynomial. Through the ***remainder theorem***, we now know that the remainder is related to evaluation of the polynomial at the point x = a. We are then led to the ***factor theorem***.

Definition

FACTOR THEOREM
If *P(a) = 0, then x – a is a factor of P(x).*
Conversely, if x – a is a factor of P(x), then P(a) = 0.

What Is It

You have already learned the difference between the Remainder Theorem and the Factor Theorem. Now, you will learn how to use these theorems to solve problems.

Example 1. Find the remainder when $x^3 + 2x^2 - 5x + 2$ is divided by $x + 3$.

Solution: We can find the remainder in two methods: by synthetic division or by remainder theorem.

Using the Remainder Theorem:

$P(x) = x^3 + 2x^2 - 5x + 2$	At $x = -3$
$P(-3) = (-3)^3 + 2(-3)^2 - 5(-3) + 2$	Substitute x by -3
$P(-3) = -27 + 2(9) + 15 + 2$	Simplify
$P(-3) = -27 + 18 + 15 + 2$	
$P(-3) = 8$	Perform the operations

Therefore, the remainder is **8**.

-3	1	2	-5	2
		-3	3	6
	1	-1	-2	**8**

We can check the answer using the Synthetic Division:

Example 2. Determine whether or not $x + 2$ is a factor of $x^3 - 2x^2 - 5x + 6$.

Solution: By definition of Factor Theorem, $x + 2$ is a factor of $x^3 - 2x^2 - 5x + 6$ if and only if the remainder is zero. Again, we can use two methods in finding the remainder.

Using the Remainder Theorem:

$P(x) = x^3 - 2x^2 - 5x + 6$	At $x = -2$
$P(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$	Substitute x by -2
$P(-2) = -8 - 2(4) + 10 + 6$	Simplify
$P(-2) = -8 - 8 + 10 + 6$	
$P(-2) = 0$	Perform the operations

Since the remainder is zero, **$x + 2$ is a factor of $x^3 - 2x^2 - 5x + 6$**.

Using the Synthetic Division:

-2	1	-2	-5	6
		-2	8	-6
	1	-4	3	**0**

Note: It is not required to present the solution in two methods. Use the method that you are most comfortable with. However, if you feel uncertain with your answer, you may use the two methods

Example 3. Show that x – 4 is a factor of $P(x) = x^3 - 6x^2 + 5x + 12$.

Solution: Since x – 4 is a factor of $P(x) = x^3 - 6x^2 + 5x + 12$, the remainder must be zero.

Using the Remainder Theorem:

$P(x) = x^3 - 6x^2 + 5x + 12$	At x = 4
$P(4) = (4)^3 - 6(4^2) + 5(4) + 12$	Substitute x by 4
$P(4) = 64 - 6(16) + 20 + 12$	Simplify
$P(4) = 64 - 96 + 20 + 12$	
$P(4) = 0$	Perform the indicated operations

The remainder is 0.

Example 4. Find k so that x + 5 is a factor of $P(x) = x^3 + 5x^2 - kx - 20$.

Solution: Since x + 5 is a factor of $P(x) = x^3 + 5x^2 - kx - 20$, the remainder is zero.

Using the Remainder Theorem:

$P(x) = x^3 + 5x^2 - kx - 20$	At x = -5
$P(-5) = (-5)^3 + 5(-5)^2 - k(-5) - 20$	Substitute x by -5
$0 = (-5)^3 + 5(-5)^2 - k(-5) - 20$	Change P(-5) by 0 (remainder is 0)
$0 = -125 + 125 + 5k - 20$	Simplify
$0 = 5k - 20$	
$5k = 20$	Solve for k
$k = 4$	Divide both sides by 5

So that x + 5 is a factor of $P(x) = x^3 + 5x^2 - kx - 20$, **k = 4**.

What's More

Exercise 1. Answer what is asked.

A. Use the Remainder Theorem or synthetic division to find each function value.

1. $f(x) = x^3 - 5x^2 - 7x + 4$

   a) f(1) b) f(-2) c) $f\left(\frac{1}{2}\right)$

2. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

   a) g(2) b) g(3) c) g(-1)

3. $h(x) = 2x^3 - 7x + 3$

   a) h(-3) b) h(5) c) h(-10)

B. Solve.

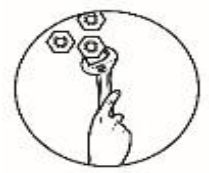
4. Determine if x – 3 is a factor of $P(x)$ where $P(x) = x^4 - 3x^3 - x + 3$.
5. Determine if x – 1 is a factor of $P(x)$ where $P(x) = x^{25} - 4$.
6. Find k so that x – 2 is a factor of $P(x) = x^3 - kx^2 - 4x + 20$.

Exercise 2. Answer is asked.

1. Use the remainder theorem to find P(2) in $P(x) = x^4 + 4x^3 - x^2 - 16x - 4$.
2. Prove that y – 3 is a factor of $3y^3 - 7y^2 - 20$ using the remainder theorem.
3. Evaluate P(4) in $P(y) = 3y^3 - 7y^2 - 20$.
4. Use the factor theorem to determine whether x – 1 is a factor of $P(x) = x^2 + 2x + 5$.
5. Use the remainder theorem to find the remainder R in $(3x^2 + 5x^3 - 8) \div (x - 4)$.

What I have learned

1. How will you determine that x –r is a factor of P(x)?
2. In the remainder theorem, what will you substitute in the polynomial expression?
3. What are the two ways on how to find the remainder when P(x) is divided by x – r?



What I can do

A. Fill the blanks to complete the statement. Write your answer in the separate sheet of pad paper.

   1. x – r is a factor of P(x) if and only if the remainder R when P(x) ÷ (x – r) is _____.
   2. By the Remainder Theorem, R = 0 if and only if _____.
   3. Thus, x – r is a factor of P(x) if and only if _____.

B. Solve this problem correctly: The volume of a rectangular solid is ($x^3 + 3x^2 + 2x$) cubic cm, and its height is (x+1) cm. What is the area of the base?

Assessment

DIRECTION: Let us determine how much you have learned from this module. Read and understand each item, then choose the letter of your answer and write it on your answer sheet.

1. What is the remainder if $x^2 - 7x - 4$ is divided by $x - 2$?

   a) 4 b) -14 c) 2 d) -2

2. Which of the following statements is true?

   a) The quotient multiplied by the dividend plus the remainder equals the divisor.
   b) If $x^2 + 5x + 7$ is divided by $x + 2$, the remainder is not 1.
   c) If the remainder is 0, then the dividend is a factor of the divisor.
   d) The quotient is a factor of the dividend if the remainder is 0.

3. Which is the remainder if $2x^3 + 4x^2 - x + 7$ is divided by $x - 2$?

   a) 35 b) 36 c) 37 d) 38

4. Is $x - 3$ a factor of $x^4 - 3x^3 - x + 3$?
   a) false b) true c) uncertain d) invalid

5. Find another factor of $x^3 - 7x^2 + 4x - 28$ if $x - 7$ is a factor.

   a) $x^2 + x + 4$ b) $x^2 - 2x - 4$ c) $x^2 - x - 4$ d) $x^2 + 4$

6. Determine which of the following binomials is a factor of $P(x) = x^3 - 7x + 5$.

   a) $x - 1$ b) $x + 1$ c) $x + 2$ d) none of these

7. Is the first polynomial a factor of the second polynomial, $x - 1$; $x^2 + 2x + 5$?

   a) yes b) no c) uncertain d) invalid

8. Which is the missing factor in the equation $x^2 - 4 = (x - 2)$ (_____)?
   a) $x - 2$ b) $x + 2$ c) $x + 4$ d) $x - 4$

9. Which of the following is a factor of $x^3 + 5x^2 - x - 5$?

   a) $x + 1$ b) $x + 5$ c) $x - 5$ d) none of these

10. What are the factors of $x^2 - 2x - 24$?

a) (x + 4) (x – 6) b) (x -8) (x + 3)

c) (x -12) (x+ 1) d) (x + 12) (x – 12)

11. What is the remainder when $x^3 - 4x^2 + x + 8$ is divided by x – 2?

a) 1 b) 2 c) 3 d) 4

12. When P(x) is divided by x – r and is equal to 0, it means _____.

a) x – r is a factor of P(x) c. P(x) is a factor x – r

b) P(x) is a factor of x – r d) x- r is the only factor of P(x)

13. Evaluate the polynomial $x^3 + x^2 + x + 3$ at the given value of x = -2

a) 3 b) -3 c) 17 d) -17

14. Which polynomial gives a remainder of zero when divided by 3x – 2?

a) $12x^2 + 15x - 18$ c) $12x^2 + 19x - 18$

b) $12x^2 + 18x + 7$ d) $12x^2 + 8x + 7$

15. Which of the following is a factor of $2x^2 - 5x + 3$?

a) x – 3 b) 2x + 3 c) x – 1 d) 3x + 2

Additional Activity

DIRECTION. Perform each given task.

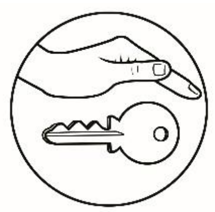
Task A. Let $P(x) = x^3 - 64$

a. Find P(4).
b. Find the remainder when P(x) is divided by x – 4.

Task B. Let $P(x) = x^3 - 64$

a. Find P(-4).
b. Find the remainder when P(x) is divided by (x + 4).

Task C. Compare your answers. What do you observe?



Answer Key

What I Know

1. d	4. c	7. a	10. c	13. b
2. b	5. a	8. b	11. a	14. b
3. b	6. b	9. c	12. b	15. d

What's More

Exercise 1

(A)

1) a = - 3 b = - 10 $c = -\frac{5}{8}$

2) a = 175 b = 1695 c = 7

3) a = -30 b = 218 c = -927

(B)

4) x – 3 is a factor

5) x – 1 is not a factor

6) k = 5

Exercise 2

1) 8 3) Not

2) Not 4) 360

3) 60

What I Can Do

(A) 1) Zero 2) Zero 3) Zero (B) $x^2 + 2x$

Assessment

1. b	4. b	7. b	10. a	13. b
2. d	5. d	8. b	11. b	14. c
3. c	6. d	9. a	12. a	15. c

Additional Activity

Task A	Task B	Task C
a) 0	a) -128	The remainder and the value of the polynomial at x – r are equal.
b) 0	b) -128	

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